SHELL MOTION UNDER AXIAL DETONATION

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A numerical solution is presented of the problem of one-dimensional motion of an incompressible cylindrical shell under axial detonation of an explosive charge. The shell strength is not taken into account. The detonation products are a polytropic gas with index k = 3. Laws of shell motion are obtained for different relationships between the charge and shell masses.

The motion of the gas and the shell is one-dimensional in the case of excitation of a detonation simultaneously along all the axes of an infinitely long charge; the diagram of the process is shown in Fig. 1. At the time (t'_*) the detonation wave 1 is incident on the shell 5 a reflected shock 3 originates in the detonation products, which converges to the axis of symmetry. After reflection from the axis, the shock 4 again emerges on the shell communicating an additional impulse thereto; the number 2 indicates the boundary of the rest zone in the figure.

The governing constants of the explosive material are the initial density ρ_0 and the detonation velocity D. The initial radius of the charge is r_0.

The system of equations for a polytropic gas in dimensionless variables is

$$\begin{gathered}
\frac{\partial \rho'}{\partial t'} + \frac{\partial (\rho' v')}{\partial r'} + \frac{\rho' v'}{r'} = 0 \\
\frac{\partial v'}{\partial t'} + v' \frac{\partial v'}{\partial r'} + \frac{1}{\rho'} \frac{\partial p'}{\partial r'} = 0 \\
\frac{\partial p'}{\partial t'} + v' \frac{\partial p'}{\partial r'} + k p' \frac{\partial v'}{\partial z'} + k \frac{p' v'}{r'} = 0
\end{gathered} \tag{1}$$

The dimensionless variables here are

$$p' = p/\rho_0 D^2, \quad \rho' = \rho / \rho_0$$
$$v' = v / D, \quad c' = c / D, \quad r' = r / r_0, \quad t' = Dt/r_0.$$

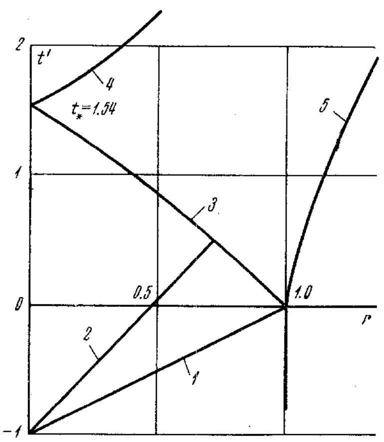

Fig. 1

The primes are henceforth omitted.

The shell is assumed incompressible, not to possess strength and to be sufficiently thin, which permits writing the boundary condition on the shell as

$$M dv / dt = Sp$$

where M, S are, respectively, the shell mass and the area of its inner surface per unit length of the charge.

The boundary condition for the detonation products on the axis is evidently v = 0. The relationship between the masses of the explosive m and the shell is characterized by the coefficient $\mu = m/M$.

The initial condition of the problem, the self-similar distribution of parameters behind the front of the diverging detonation wave,

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was found by integrating the system (1) for constant entropy (see [1]). Values of the dimensionless detonation variables (at the Chapman–Jouguet point) are determined according to [2] as

$$\rho_{C-J} = \frac{1}{k+1},\ p_{C-J} = \frac{k+1}{k},\ v_{C-J} = \frac{1}{k+1},\ c_{C-J} = \frac{k}{k+1}$$

The isentrope of the detonation products has the form $p = k^3(k+1)^{-4}\rho^3$. It is henceforth assumed that $k = 3$.

The radius of the rest zone is $r_r = 0.485$, the relative mass and energy in the rest zone are 0.197 and 0.117, respectively; the density is $\rho_r = 0.84$; and the pressure is $p_r = 0.0625$. The internal and kinetic energies of the detonation products with cylindrical symmetry are 0.897 and 0.103 in relative units, respectively.

Introducing the space variable $\xi = r/R$ (R is the internal running radius of the shell), we transform the system (1) to moving Eulerian coordinates

$$\begin{aligned} &\frac{\partial\rho}{\partial t} + A\frac{\partial\rho}{\partial\xi} + B\rho\frac{\partial v}{\partial\xi} = \rho C \qquad \left(A = \frac{v-\xi R'}{R}\right)\\ &\frac{\partial v}{\partial t} + A\frac{\partial v}{\partial\xi} + \frac{B}{\rho}\frac{\partial p}{\partial\xi} = 0 \qquad \left(B = \frac{1}{R}\right)\\ &\frac{\partial p}{\partial t} + A\frac{\partial p}{\partial\xi} + Bp\frac{\partial v}{\partial\xi} = pC \qquad \left(C = -\frac{v}{R}\right)\end{aligned} \tag{2}$$

A finite-difference approximation of the system (2) is performed by using an explicit scheme of second-order accuracy (a modified Lax-Wendroff scheme [3]). The computation is carried out in two stages. Values of the desired functions are obtained on a layer with time index $n + a$ (n is the layer on which values of all the desired functions are known, and $a > 0$) at points with half-integer space index in the first stage. In the second stage the derivative dw/dt is approximated at the point $(n + {}^1/_2,\ m)$ by the difference $(w_m^{n+1} - w_m^n)/\Delta t$, and the derivative $dw/d\xi$ is approximated by the linear combination

$$\alpha\,(w_{m+1/2}^{n+a} - w_{m-1/2}^{n+a})/\Delta\xi + \beta\,(w_{m+1}^{n} - w_{m-1}^{n})/\Delta\xi$$

Here α and β are coefficients obtained $(n + {}^1/_2,\ mh)$ during linear interpolation of the function w at the points (nt, mh) and $[(n + a)t,\ mh)$. Values of the coefficients are evaluated at this same point.

The parameters on the shell are computed by the following formulas:

$$\rho_N^{n+1} = [2\Delta t\,(\rho C)_{N-1/2}^{n+1/2} + (1 - \tau A_{N-1/2}^{n+1/2})(\rho_N^n - \rho_{N-1}^{n+1}) + (1 + \tau A_{N-1/2}^{n+1/2})\,\rho_{N-1}^n$$
$$- \tau\,(\rho B)_{N-1/2}^{n+1/2}(u_N^{n+1} - u_{N-1}^{n+1} + u_N^n - u_{N-1}^n)]/(1 + \tau A_{N-1/2}^{n+1/2}),\quad \Delta t/\Delta\xi = \tau$$

The pressure p is evaluated by an analogous formula

$$u_N^{n+1} = u_N^{n+8/3}\,\Delta t\mu\, p^{n+1/2}\,R^{n+1/2}/k$$

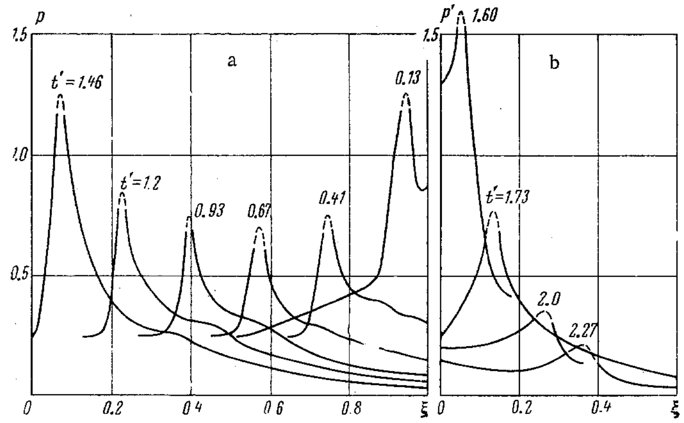

Fig. 2

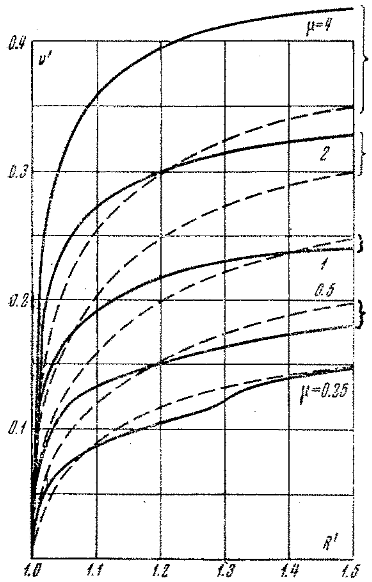

Fig. 3

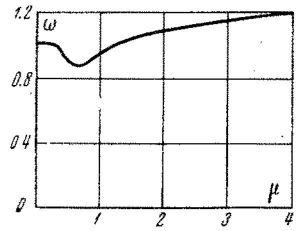

Fig. 4

The parameters at the center are computed by the following formulas:

$$\rho_0^{n+1} = [2\Delta t\,(\rho C)^{n+1/2}_{1/2} + (1 + \tau A^{n+1/2}_{1/2})(\rho_0{}^n - \rho_1^{n+1}) + (1 - \tau A^{n+1/2}_{1/2})\,\rho_1{}^n - (\rho B)^{n+1/2}_{1/2}\tau\,(u_1^{n+1} - u_0^{n+1} + u_1{}^n - u_0{}^n)] / (1 - \tau A^{n+1/2}_{1/2})$$

The pressure p_0^{n+1} is calculated by an analogous formula

$$u_0^{n+1} = 0$$

The constraint

$$\Delta t \leqslant KR\Delta\xi / \max(|v| + c)$$

was imposed in order to assure stability of the computation at the time spacing.

Here max (|v| + c) is the maximum sum of the absolute value of the mass flow rate and the local speed of sound on the layer. By definition of the dimensionless parameters $c = {}^9\!/_{16}\rho$. The quantity K agrees with the Courant number, where K is assumed to equal 0.4 in the numerical computations. The computation was hence stable.

The system (2) was integrated for the values μ = 0.25, 0.5, 0.75, 1.0, 2.0, 2.5, 4.0.

A check on the correctness of the computation was performed by using a calculation of the total system energy integral on each time layer. The maximum ΔE did not exceed 3.8% during the computation. The correctness of the computation was also checked by verifying the Hugoniot condition on the reflected front for waves of different intensity.

The pressure distribution in the detonation products at the dimensionless coordinate ξ = r/R at different times is shown in Fig. 2 (a, prior to reflection by the axis of symmetry; b, after reflection). That the reflected wave profile is somewhat nonmonotone is explained by singularities in the finite difference scheme.

The greatest errors in determining the quantities originate at the time of wave reflection from the shell and the axis of symmetry. The pressure of detonation wave reflection from the shell fluctuated between 2.20-2.60 for a theoretical value of 2.39 [2]. Since the reflected wave was weak, the entropy growth therein did not exceed 1.5-2.0% on the average. A computation of one of the versions (μ = 1.0) in an isentropic approximation showed that the discrepancy in shell velocities does not exceed 0.5%.

As the reflected wave converges to the axis of symmetry the wave amplitude theoretically tends to infinity. To assure stable operation of the scheme, the frontal pressure was smoothed, which could not however result in substantial errors since the reflected wave energy is comparatively slight. A check of the total system energy showed that as the shock passed through the center its magnitude diminished by less than 0.1%.

Laws for the one-dimensional motion of a shell are compared in Fig. 3 for the case of axial detonation (solid lines) and detonation products initially at rest (dashed lines). It has been assumed that the solidity of the shell is not spoiled up to R' = 1.5 ($R' = R/r_0$). As should have been expected, the axial detonation assures considerably greater initial shell accelerations.

Let us introduce the ratio

$$\omega_R = (v_1 / v_2)_R = f(\mu, R)$$

Here v_1, v_2 are the shell velocities at a given radius R' under axial detonation and initially in a gas at rest, respectively. A graph of the dependence $\omega_{(1.5)} = f(\mu)$ for a fixed value of R' = 1.5 is presented in Fig. 4.

The initial stage of the acceleration plays a decisive role in collection of the velocity in the $\mu > 1.25$ case; consequently, $\omega_{(1.5)} > 1$. The limit value of ω evidently holds at $\mu \to \infty$, where this value is independent of the quantity R' and equals $D/c_0 = 1.64$ [$c_0 = D(^3/_8)^{1/2} = 0.61D$ is the speed of sound in the detonation products at rest].

For $\mu = 0.25$–1.25 the initial period of acceleration (up to equilibration of the velocities $\omega = 1$) terminated at R' = 1.1–1.4. Because of entrainment of part of the energy in the reflected wave the value is $\omega_{(1.5)} < 1$. Finally, in the case $\mu < 0.25$, the reflected wave succeeds in reaching the shell at low values of R', which results in equilibration of the velocities.

Evidently, ω depends essentially on the quantity R' also. For example, for $\mu = 2$ and R' = 1.5, 1.2, 1.1 the index ω is 1.09, 1.21, 1.34, respectively.

The minimal pressure in the detonation products occurring at the shell for $\mu = 4$, R' = 1.5 was 0.005, which for a frontal pressure of p_{C-J} = 200–300 kbar will correspond to p = 1–1.5 kbar. This circumstance permits considering the assumption k = 3 acceptable since, according to [2], the pressure at the point of transition from isentropy $p = \alpha\rho^3$ to nonisentropy $p = b\rho^\gamma$ ($\gamma = 1.2$–1.4) is 1.5–2.0 kbar.

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